

Máster in Mathematical Engineering

Course: Hilbert spaces, Wavelets and sampling theory

Teacher: Antonio García García

Weekly planning: Each weekly session has a duration of 3 hours including theory and solving of exercises.

1. **Week 1:** Remind on the Hilbert spaces theory: orthogonal projection; basic operator theory.
2. **Week 2:** Orthonormal bases in a separable Hilbert space: examples.
3. **Week 3:** Classical Fourier series: pointwise convergence.
4. **Week 4:** Riesz bases and Frames.
5. **Week 5:** Frame theory (continuation).
6. **Week 6:** Fourier transform in $L^1(\mathbb{R})$. Poisson summation formula.
7. **Week 7:** Fourier transform in $L^2(\mathbb{R})$. Hilbert transform in $L^2(\mathbb{R})$
8. **Week 8:** Reproducing kernel Hilbert spaces.
9. **Week 9:** Sampling theory in Paley-Wiener spaces: generalizations.
10. **Week 10:** Sampling theory in Shift-invariant subspaces of $L^2(\mathbb{R})$.
11. **Week 11:** Time-frequency analysis: Continuous Gabor and Wavelet transforms. Discretization.
12. **Week 12:** Orthonormal bases of wavelets for $L^2(\mathbb{R})$ obtained from a Multiresolution Analysis.
13. **Week 13:** Decomposition and reconstruction algorithms: interpretation in terms of digital filters.
14. **Week 14:** Examples of orthonormal bases of wavelets for $L^2(\mathbb{R})$.